DUALISM WAVE – PARTICLE AND PRINCIPLE OF RELATIVITY

Pietro Di Mauro*
(Liceo Scientifico "E. Fermi" – Paternò (CT)
Gruppo di lavoro Mondotre – La Scuola Italica,
Associazione Culturale " Salvatore Notarrigo")
*work partially supported by INFN - Sez. di Catania

ABSTRACT: Is it compatible the de Broglie's relation with the principle of relativity? A few reflections and ideas to put together the principle of relativity, foundation of classic and relativistic physics, and the wave – particle dualism, basis of quantum mechanics.

Key words: dualism wave – particle, principle of relativity,
de Broglie.

1. Introduction

Landé remarked in some of his works[1] that the de Broglie relation

$$p = \frac{h}{\lambda} \qquad (1)$$

is not covariant with the principle of relativity: the momentum of a particle changes if measured in another system that moves relative to the first translational motion with uniform speed u (we always consider it, for simplicity, positive), while the wavelength associated with the de Broglie relation should not change ("*A snapshot of ocean waves taken from a lighthouse displays the same wavelength as one taken from an airplane*"[2]). That is, in the two systems it would be $\lambda' = \lambda$ and

$$p' = p + mu \qquad (2)$$

and this is incompatible with the relation given by de Broglie, because there should be even $p' = p$, given that $\frac{h}{\lambda'} = \frac{h}{\lambda}$. "*In the nonrelastivistic domain, the relation (1) violates one of the most sacred postulates of theoretical physics, that of covariance: whereas a momentum $p = mv$ of particle in a system O changes to $p' = p + mu$ in a system O' moving with velocity u relative to O, a wavelength λ remains unchanged, $\lambda' = \lambda$* [3]."

2. Considerations and ideas

In the context of relativistic paradigm in which, however, the de Broglie relation was deduced, as we will see later, you would get the same by using the formulas provided by special relativity. In this case not only have we to take account of the new definition of the momentum but we can also consider, consistently, the so-called contraction of lengths applied to the wavelength and the relativistic rule of speed addition. So next

$$p = \gamma m_0 v = \frac{h}{\lambda} \qquad (3)$$

you will have to write even

$$\lambda' = \frac{\lambda}{\gamma'} \qquad (4)$$

being $\gamma' = \frac{W}{\sqrt{1 - W^2/c^2}}$ with $W = \frac{v + u}{1 + v \cdot u/c^2}$. Following the same proceedings as before, for the other system we would have

$$p' = \gamma' m_0 W \qquad (5)$$

united to another expression given by

[1] A. Landè: "*Quantum fact and fiction. IV*", Am. J. Phys., 43 (8), 1975, pp. 701 – 704; "*The decline and fall of quantum dualism*", Philosophy of Science, 38 (2), 1971, pp. 221 – 223.
[2] A. Landè, op. cit., p. 702, p. 222.
[3] A. Landè, op. cit., p. 702, p. 222.

$$p' = \frac{h}{\lambda'} = \frac{\gamma' \cdot h}{\lambda} = \gamma' \cdot p \,. \qquad (6)$$

Substituting the used relationships we obtain

$$m_0 W = p \,, \qquad (7)$$

valid for only one value $u = \dfrac{v(\gamma - 1)}{1 - v^2 \cdot \gamma / c^2}$.

In my earlier work on the de Broglie relation[4], I pointed out some difficulties in the derivation of the same as in the experimental verification of what it is believed to be the relationship that has extended the dual wave - particle also matter.

Equality $h\nu_0 = m_0 c^2$, on the basis of the de Broglie relation, a synthesis of the two major theories that emerged in the science of the last century, I pointed out that it is just incompatible with special relativity: for a particle in motion by increasing speed also the energy and the frequency should increase against time dilation, inverse of the frequency, predicted by Einsteinian theory!

Furthermore it brings with it another inconsistency.

In his Nobel Lecture[5] de Broglie derives his famous relation simply by equalities

$$p = \gamma m_0 v = \frac{h\nu}{c^2} v = \frac{h\nu}{V} = \frac{h}{\lambda} \qquad (8)$$

By using the concept, all relativistic, of "total energy" $E = \gamma m_0 c^2$ of the particles (with γ the usual factor Lorentzian $\left(1 - v^2/c^2\right)^{-1/2} = \left(1 - \beta^2\right)^{-1/2}$) and, in this case, also the wave phenomenon associated with them. But, while the amount of motion for a particle at rest, as regards to a certain observer, is nothing, and this would lead to a wavelength of de Broglie tending to infinity, its total energy is different from zero being still valid $E = m_0 c^2$. This would lead to even associate a wave to a stopped particle, against precisely the relation given by de Broglie.

If we use in the deduction made by de Broglie instead of total energy the kinetic one in the Planck-Einstein relation $h\nu = m_0 c^2 (\gamma - 1)$ (which is zero if it is the momentum) we would obtain, following the same steps[6], instead of the de Broglie relation, the equality

$$p = \frac{h}{\lambda} + m_0 v \,. \qquad (9)$$

Clearly, if the found out relationship, could be considered a new definition of momentum which contains the two corpuscular and wavelike aspects, it would be covariant according to what specified by Lande, with the principle of relativity, assuming its $\lambda' = \lambda$. In fact we will have

$$p' = \frac{h}{\lambda'} + m(v + u) = \frac{h}{\lambda} + mv + mu = p + mu \qquad (10)$$

i.e. the (2).

But exactly how it was obtained, being $p = \gamma m_0 v$, i.e. the so-called relativistic momentum, it should be stated, first of all, that the relativistic increase, compared to the classical momentum , is given just by the wavelike aspect supplied by the term containing the length of wave. By the same procedure done previously for the usual relationship of de Broglie, using the same symbols with the same definitions as before, we will have to write

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[4] P. Di Mauro, "*Sulla relazione di de Broglie*", in Atti del XLVII Congresso Nazionale AIF, Roma 2008 - La Fisica nella Scuola, Bollettino trimestrale dell'AIF, Anno XLII – Supplemento al n. 3 luglio – settembre 2009, pp. 68 – 73, Monotipia Cremonese (CR). ISSN: 1120 – 6527.

[5] L de Broglie, "*The wave nature of the electron*" Nobel Lecture, December 12, 1929, pag. 247.

[6] P. Di Mauro, op. cit. p. 70.

$$\begin{cases} p = \dfrac{h}{\lambda} + m_0 v \\ p' = \dfrac{\gamma' \cdot h}{\lambda} + m_0 W \end{cases} \qquad (11)$$

only possible for $W - W\sqrt{1 - W^2/c^2}$ given!

Coherently with the ideas of de Broglie we can define the frequency of the wave associated with a moving particle as

$$\nu = \frac{c^2}{\lambda \cdot v} = \frac{V}{\lambda}. \qquad (12)$$

We try to find out how the defined frequency changes when the particle is observed in the motion system with velocity u as regards to the system fixed to the particle. In other words, we have to consider a kind of Doppler effect also for the waves associated with particles.

We need, meanwhile, to take account of how the kinetic energy varies in a reference system in uniform translatory motion as to the integral one with the particle. This shows a problem. The kinetic energy supplied to a body by some force in a certain reference system becomes larger when measured according to another reference system in uniform translatory motion as to the first without having, indeed, an increase in the work done by force. In fact, using the classical expression of the kinetic energy we have

$$E' = E + p \cdot u + \frac{1}{2} m_0 u^2 \qquad (13)$$

while with the relativistic we obtain is

$$E' = E \frac{\gamma' - 1}{\gamma - 1}. \qquad (14)$$

Similarly, for the total energy it occurs

$$E' = E \frac{\gamma'}{\gamma}. \qquad (15)$$

The frequency measured in the moving system must satisfy the definition given before (12), and we can write likewise

$$\nu' = \frac{c^2}{\lambda' \cdot W}. \qquad (16)$$

It must immediately be noted that by these definitions the wavelength cannot remain unchanged because with increasing speed *v* the frequency decreases, against just what one would expect from a similar Doppler effect for the waves associated with particles. However, by replacing therein the de Broglie relation is obtained

$$\nu = \frac{\gamma m_0 c^2}{h} \qquad (17)$$

(the starting point of de Broglie) and

$$\nu' = \frac{\gamma' m_0 c^2}{h}. \qquad (18)$$

Instead, if we were to use the kinetic energy, with the momentum given by (7), we would have

$$\nu = \frac{m_0 c^2 (\gamma - 1)}{h} \text{ and } \nu' = \frac{m_0 c^2 (\gamma' - 1)}{h}. \qquad (19)$$

We have

$$\nu' = \nu \frac{\gamma'}{\gamma} \text{ and } \nu' = \nu \frac{\gamma' - 1}{\gamma - 1} \qquad (20)$$

by comparing them respectively to the two forms of energy considered. Replacing them in the definition of a given frequency, and taking into account the relativistic contraction of the wavelength, we obtain in the first case

$$W = \gamma v \text{ and } W = \frac{v\gamma'(\gamma - 1)}{\gamma' - 1} \qquad (21)$$

in the second case. In both cases, the fixed velocity *v*, the equalities will provide us with constant values of *u* that verify them. And this against the arbitrariness of the velocity *u* of the considered system! Therefore, by examining the thing from also the energy point of view, the de Broglie relation remains incompatible, in whatever its form, with the principle of relativity and the usual definitions of momentum.

3. Conclusions

The various formulas we worked out constantly remind us the fundamental idea that in science the models with which you want to work are more important than theoretical formalizations! And in this case the model used, the wave associated with a particle, presents more than one problem. The particle, which is the unit, located in a point in space, is "distributed" in a wave characterized by its wavelength. It is the same de Broglie who postulates an oxymoron. In such a way de Broglie says in his Nobel Lecture:

> "*I thus arrived at the following overall concept which guided my studies: for both matter and radiations, light in particular, it is necessary to introduce the corpuscle concept and the wave concept at the same time. In other words the existence of corpuscles accompanied by waves has to be assumed in all cases. However, since corpuscles and waves cannot be independent because, according to Bohr's expression, they constitute two complementary forces of reality, it must be possible to establish a certain parallelism between the motion of corpuscle and the propagation of the associated wave*"[7].

The mingling of the two models is placed just on the basis, as we have seen, of all deductions thus reversing the role between physics and mathematics, the latter, not coming from definite elementary physical operations, can never have a "*real*" meaning but only a "*formal*" one.

So the de Broglie relation, widely regarded as essential both for the synthesis that operates between quantum theory and relativity and for its fundamental role in modern quantum mechanics, shows some uncertainties, as it was deduced, for its compatibility with the principle of relativity (which is also a fundamental postulate of Einstein relativity!) and, not the least, for its experimental verification.

But on the other hand it is the same de Broglie who concludes his doctoral thesis in 1924 with these words:

> "*I have left the definitions of phase waves and the periodic phenomena for which such waves are a realization, as well as the notion of a photon, deliberately vague. The present theory is, therefore, to be considered rather tentative as Physics and not an established doctrine*"[8].

Eye–Knowledge

De Broglie
Di Mauro
dualism wave – particle
Landè
Perucca
principle of relativity

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[7] L de Broglie, "*The wave nature of the electron*" Nobel Lecture, December 12, 1929, p. 247.
[8] L. de Broglie, op. cit. p. 72.